Course *Statistics for Stochastic Processes*
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Exercises: sheet 2

Oral presentation (By 09.11.2017)

Exercise 1. *Let X be a centered Gaussian variable with variance 1 and let U be a Bernoulli random variable with parameter 1/2, independent of X.*

- *Show that $Y = X\mathbf{1}_{\{U=1\}} - X\mathbf{1}_{\{U=0\}}$ is a a centered Gaussian variable with variance 1.*
- *Show that $Cov(X, Y) = 0$ but X and Y are not independent.*
- *Derive from it a process that is a white noise but not a sequence of i.i.d random variables.*

Exercise 2. *Consider the function γ defined as*

$$\gamma(h) = \begin{cases} 1 & \text{if } h = 0, \\ \rho & \text{if } |h| = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Show that γ is an autocovariance function if and only if $|\rho| \leq 1/2$. Give an example of a weakly stationary process having γ as autocovariance function.

Exercise 3. *Let $X = (X_t)_{t\in\mathbb{R}}$ be a weakly stationary process. Construct an unbiased estimator of $\mu = \mathbb{E}[X_t]$ based on observing $(X_t)_{t\in[0,T]}$.*

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Exercise 4. *Prove that the sum of two weakly stationary processes is not always a weakly stationary process. [**Hint:** Consider $Z_t + (-1)^t Z_{t-1}$, where $Z \sim WN(0, \sigma^2)$].*

Exercise 5. *Consider the equation*

$$X_t - \phi X_{t-1} = Z_t + \theta Z_{t-1}, \quad t \in \mathbb{Z} \tag{1}$$

where $Z \sim WN(0, \sigma^2)$ and $\phi, \theta \in \mathbb{R}$.

- *Under which conditions on ϕ and θ, does a weakly stationary solution X of (1) exist?*

- *Under which conditions the weakly stationary solution is causal?*

In the following, it will be assumed that such conditions are verified.

- *Give a representation of the solution* X *as a series* $\sum_{k\in\mathbb{Z}} \psi_k Z_{t-k}$.
- *Justify the convergence of this series, specify in what sense it converges and give a formula for the coefficients* $(\psi_k)_{k\in\mathbb{Z}}$.
- *Compute the autocovariance function of* X.